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Restudy of the structures and interactions of the soliton in the asymmetric Nizhnik–Novikov–Veselov equation

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Abstract
Some new exact solutions of the $(2+1)$-dimensional asymmetric Nizhnik–Novikov–Veselov equation are presented using the bilinear method. The solutions to describe the interactions between two dromions, between a line soliton and a y-periodic soliton, and between two y-periodic solitons are included in our results. The detailed behaviour of the interactions is illustrated both analytically and graphically. Our analysis shows that the forms of soliton solutions and interacting properties between two solitons are related to the forms of the parameters and interaction constants.

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1. Introduction

The recent development of nonlinear wave theory clarifies the role of the 'soliton' in various systems [1]. Solitons are stable and the interactions between them affect only the phase shifts. Therefore, solitons are regarded as the fundamental structures in nonlinear integrable systems. The spatial structures of solitons are usually the solitary waveforms whose amplitudes tend to zero as $x \longrightarrow \pm\infty$, or the kink forms whose amplitudes tend to two different constants as $x \longrightarrow \pm\infty$. The soliton structures and their properties of $(1+1)$-dimensional integrable nonlinear evolution equations have been very well understood. However, the soliton structure in higher spatial dimensions continues to be much more intricate. Recently, since the pioneering work of Boiti *et al* [2], the study of the exponentially localized soliton solutions called dromions has been attracting the attention of physicists and mathematicians. Usually, dromion solutions are driven by two or more nonparallel straight-line ghost solitons [3, 4]. There also exist some dromion solutions of the physical fields for one type of nonlinear models such as the Davey–Stewartson (DS) and Nizhwik–Novikov–Veselov (NNV) [5]. However, for other types of equations such as the Kadomtsev–Petviashvili (KP) and the breaking soliton equations, the dromion solutions exist only for some suitable potential of the fields [6].

In this paper, we are interested in the structures and interactions of the soliton for $(2+1)$-dimensional integrable systems. It is well known that the soliton interactions of $(1+1)$-dimensional integrable models are elastic. This means that there is no exchange of energy (no changes of shape and velocity) among interacting solitons. However, some different results have been reported for $(2+1)$-dimensional integrable systems. Consequentially, the dromion interactions are inelastic for the DS equation [7], but for the Nolacal–Bussinesq (NLBQ) equation and Sawada–Kotera (SK) system they are elastic [8, 9]. Here, we hope to understand the reasons why the interaction between dromions is elastic for some models and inelastic for others. Because it is difficult to solve higher-dimensional nonlinear models, it becomes difficult to reveal the structures and properties of interaction of the soliton in higher-dimensional models. There is some wealth of methods for finding special solutions of higher-dimensional integrable models. Two important methods are the bilinear method and variable separating approach. However, there are some limitations when we deal with some concrete problems using the two methods. For example, for the asymmetry NNV (ANNV) equation, some solutions, which are used to describe the interactions between two dromions for physical field u, between a line soliton and a y-periodic soliton, and between two y-periodic solitons for physical field v, cannot be obtained by the standard Hirota bilinear method because of the forms of the interaction constants. Even using the variable separating approach, we cannot obtain the solution to describe the interaction between two y-periodic solitons due to the complexity of the solution.

The bilinear method is regarded as a good method to get soliton structures. However, some coefficients in the multi-soliton solution form presented by Hirota may be free coefficients. With the help of the bilinear method, after assuming the forms of solution according to the research needs, we obtain abundant soliton structures about the ANNV equation. Using these exact solutions the phenomena of interaction are discussed in detail.

The paper is organized as follows. In section 2, some exact solutions of the ANNV equation are presented with the help of the bilinear method. The interactions between two dromions, between a line soliton and a y-periodic soliton and between two y-periodic solitons are discussed in section 3, while a summary and a discussion are in section 4.

2. Exact solutions of the ANNV equation

The ANNV equation

$$u_t + u_{xxx} + 3[uv]_x = 0 \qquad u_x = v_y \tag{1}$$

may be considered as a model for an incompressible fluid where u and v are the components of the (dimensionless) velocity [10]. The spectral transformation for this system had been investigated in [5, 11]. This system may be considered as a generalization [12] to $(2+1)$ dimensions of the results from Hirota and Satsuma [13]. The non-classical symmetries, Painlevé property and similarity solutions of the system were investigated by Clarkson and Mansfield [14]. Equation (1) has bilinear form [15]

$$u = 2(\log f)_{xy} \qquad v = 2(\log f)_{xx} \tag{2}$$

$$\left(D_y D_t + D_y D_x^3\right) f \cdot f = 0. \tag{3}$$

In [16], Radha and Lakshmanan studied the dromion solution of (3). They gave two dromion solutions with the following special form,

$$u = -2(\log f)_{xy} \qquad f = 1 + \exp(\xi_1) + \exp(\xi_2) + \exp(\xi_3) + K(\exp(\xi_1 + \xi_2) + \exp(\xi_2 + \xi_3))$$
$$\xi_1 = k_1 x - k_1^3 t + c_1 \qquad x_2 = l_1 y + c_2 \qquad x_3 = k_2 x - k_2^3 t + c_3 \tag{3.1}$$

where c_1, c_2, c_3 are all constants. In [17], Lou extended the work of [16], presented the assumtion that the parameters k_i, c_i in equation (3.1) are functions of y, and obtained more abundant dromion structures. He obtained the conclusion that the dromions can be driven not only by some perpendicular line ghost solitons but also by some non-perpendicular line and curved line ghost solitons.

Although some special dromion solutions have been given by [16, 17] the interactions between two dromions have not been studied in their works. In order to study interacting phenomena between two solitons, we should find more solution structures.

It can be proved that equation (3) possesses the standard multi-soliton solution form proposed by Hirota [18]. Here, we write down the three-soliton solution expression

$$\begin{aligned} f = {} & 1 + \exp(\eta_1) + \exp(\eta_2) + \exp(\eta_3) + a_{12}\exp(\eta_1+\eta_2) + a_{13}\exp(\eta_1+\eta_3) \\ & + a_{23}\exp(\eta_2+\eta_3) + a_{12}a_{13}a_{23}\exp(\eta_1+\eta_2+\eta_3) \end{aligned} \tag{4}$$

$$\eta_i = k_i x + l_i y + \omega_i t + \eta_{i0} \qquad l_i\omega_i + l_i k_i^3 = 0 \tag{5}$$

$$a_{ij} = -\frac{A(p_i - p_j)}{A(p_i + p_j)} = -\frac{(l_i - l_j)\left(-k_i^3 + k_j^3 + (k_i - k_j)^3\right)}{(l_i + l_j)\left(-k_i^3 - k_j^3 + (k_i + k_j)^3\right)}. \tag{6}$$

If we hope to obtain the solution to describe the interaction between two dromions for field component

$$u = 2(\log f)_{xy} \tag{7}$$

from equations (4)–(6), the parameters $\{k_i, l_i\}$ $(i = 1, 2, 3)$ must be taken as $\{(k_1, 0), (k_2, 0), (0, l_3)\}$. One can easily see from the above expression that one cannot obtain the solution of two-dromion interaction for u and the solution to describe the interaction between a line soliton and a y-periodic soliton for v because of the form of a_{ij}. For the same reason, we cannot get the solution to describe the interaction between two y-periodic solitons for v.

If we assume the solution of equation (3) has the following form

$$\begin{aligned} f = {} & 1 + \exp(\eta_1) + \exp(\eta_2) + \exp(\eta_3) + a_{12}\exp(\eta_1+\eta_2) + a_{13}\exp(\eta_1+\eta_3) \\ & + a_{23}\exp(\eta_2+\eta_3) + m\exp(\eta_1+\eta_2+\eta_3) \end{aligned} \tag{8}$$

$$\eta_1 = k_1 x + \omega_1 t + \eta_{10} \qquad \eta_2 = k_2 x + \omega_2 t + \eta_{20} \qquad \eta_3 = l_3 y + \omega_3 t + \eta_{30} \qquad \omega_i = -k_i^3 \tag{9}$$

we find when

$$m = a_{12} + (a_{13} - a_{23})\frac{k_1 - k_2}{k_1 + k_2} \tag{10}$$

equation (8) with (9) is a solution of equation (3). It describes the interaction between two dromions for field component u.

If we assume that the solution of (3) possesses the form of equation (8) and

$$\begin{aligned} & \eta_1 = k_1 x + \mathrm{i}\delta_1 y + \omega_1 t \qquad \eta_2 = k_1 x - \mathrm{i}\delta_1 y + \omega_2 t \\ & \eta_3 = k_3 x + \omega_3 t \qquad a_{13} = a_{23} \qquad m = a_{12}a_{13}a_{23} \end{aligned} \tag{11}$$

then when

$$\begin{aligned} & \omega_1 = \omega_2 = -k_1^3 \qquad \omega_3 = -\frac{k_3}{k_1^2}\left(-k_3^2k_1^2 + 3k_1^4 - 3\delta_1^2\right) \\ & a_{13} = a_{23} = \frac{-2k_1^3k_3 - l_1^2 + k_1^2k_3^2 + k_1^4}{k_3^2k_1^2 + k_1^4 + 2k_1^3k_3 - l_1^2} \end{aligned} \tag{12}$$

the solution (8) with (11) and (12) is a solution to describe the interaction between a line soliton and a y-periodic soliton for field component v. It can be written as

$$v = 2(\log f)_{xx} \tag{13}$$

$$\begin{aligned} f = {} & 1 + 2\exp(\xi_1)\cos(\eta_1) + \exp(\xi_2) + a_{12}\exp(2\xi_1) \\ & + 2a_{13}\exp(\xi_1+\xi_2)\cos(\eta_1) + a_{12}a_{13}^2\exp(2\xi_1+\xi_2) \end{aligned} \tag{14}$$

where

$$\xi_1 = k_1x - k_1^3t \qquad \xi_2 = k_3x + \omega_3 t \qquad \eta_1 = \delta_1 y \qquad \omega_3 = -\frac{k_3}{2k_1^2}\left(-k_3^2k_1^2 + 3k_1^4 - 3\delta_1^2\right). \tag{15}$$

Taking the following assumption in equation (3)

$$\begin{aligned} f = {} & 1 + \exp(\eta_1) + \exp(\eta_2) + \exp(\eta_3) + \exp(\eta_4) + a_{12}\exp(\eta_1+\eta_2) + a_{13}\exp(\eta_1+\eta_3) \\ & + a_{14}\exp(\eta_1+\eta_4) + a_{23}\exp(\eta_2+\eta_3) + a_{24}\exp(\eta_2+\eta_4) \\ & + a_{34}\exp(\eta_3+\eta_4) + a_{12}a_{13}a_{23}\exp(\eta_1+\eta_2+\eta_3) + a_{12}a_{14}a_{24}\exp(\eta_1+\eta_2+\eta_4) \\ & + a_{23}a_{24}a_{34}\exp(\eta_2+\eta_3+\eta_4) + a_{13}a_{14}a_{34}\exp(\eta_1+\eta_3+\eta_4) \\ & + a_{12}a_{13}a_{14}a_{23}a_{24}a_{34}\exp(\eta_1+\eta_2+\eta_3+\eta_4) \\ \eta_1 = {} & k_1x + \mathrm{i}\delta_1 y + \omega_1 t \qquad \eta_2 = k_1x - \mathrm{i}\delta_1 y + \omega_2 t \\ \eta_3 = {} & k_2x + \mathrm{i}\delta_2 y + \omega_3 t \qquad \eta_4 = k_2x - \mathrm{i}\delta_2 y + \omega_4 t \end{aligned} \tag{16}$$

when

$$\omega_1 = \omega_2 = -k_1^3 = -\omega_{11} \qquad \omega_3 = \omega_4 = -k_2^3 = -\omega_{22} \tag{17}$$

$$a_{13} = a_{24} = \frac{(\delta_2-\delta_1)(k_2-k_1)}{(\delta_2+\delta_1)(k_2+k_1)} \qquad a_{23} = a_{14} = \frac{(\delta_2+\delta_1)(k_2-k_1)}{(\delta_2-\delta_1)(k_2+k_1)} \tag{18}$$

the solution (16) describe the interaction between two y-periodic solitons for the physical field v. It can be expressed as

$$v = 2(\log f)_{xx} \tag{19}$$

$$\begin{aligned} f = {} & 1 + a_{12}\exp(2\xi_1) + 2\exp(\xi_1)\cos(\eta_1) + a_{34}\exp(2\xi_2) + 2\exp(\xi_2)\cos(\eta_2) \\ & + 2\exp(\xi_1+\xi_2)(a_{13}\cos(\eta_1+\eta_2) + a_{23}\cos(\eta_2-\eta_1)) \\ & + 2a_{12}a_{13}a_{23}\exp(2\xi_1+\xi_2)\cos(\eta_2) + 2a_{34}a_{13}a_{23}\exp(2\xi_2+\xi_1)\cos(\eta_1) \\ & + a_{12}a_{34}a_{13}^2a_{23}^2\exp(2(\xi_1+\xi_2)) \end{aligned} \tag{20}$$

where

$$\xi_1 = k_1x - \omega_{11}t \qquad \xi_2 = k_2x - \omega_{22}t \qquad \eta_1 = \delta_1 y \qquad \eta_2 = \delta_2 y. \tag{21}$$

3. Interaction between two solitons

3.1. Interaction between two dromions

In this subsection, we consider the interaction between two dromions about field u expressed by (7) and (8) with (9) and (10). When we assume that $k_1 > 0$, $k_2 > 0$ and $\omega_2/k_2 > \omega_1/k_1$, we obtain the expressions for two separated dromions before interaction

$$u_1 = 2(\log f_1)_{xy} \tag{22}$$

$$f_1 = 1 + \exp(\eta_1) + \exp(\eta_3) + a_{13}\exp(\eta_1 + \eta_3) \tag{23}$$

$$u_2 = 2(\log f_2)_{xy} \tag{24}$$

$$f_2 = \exp(\eta_1)\left(1 + \exp(\eta_3 + \ln a_{13}) + \exp(\eta_2 + \ln a_{12}) + a_{23}\exp\left(\eta_2 + \eta_3 + \ln\frac{m}{a_{23}}\right)\right) \tag{25}$$

and the expressions for the two dromions after interaction

$$u_3 = 2(\log f_3)_{xy} \tag{26}$$

$$f_3 = \exp(\eta_2)\left(1 + \exp(\eta_1 + \ln a_{12}) + \exp(\eta_3 + \ln a_{23}) + a_{13}\exp\left(\eta_1 + \eta_3 + \ln\frac{m}{a_{13}}\right)\right) \tag{27}$$

$$u_4 = 2(\log f_4)_{xy} \tag{28}$$

$$f_4 = 1 + \exp(\eta_2) + \exp(\eta_3) + a_{23}\exp(\eta_2 + \eta_3). \tag{29}$$

Taking into account that u is unchanged even if f is multiplied by $\exp(ax + b)$ with a and b independent of x, we have only to consider the form of f. From (22)–(29), one can see that if $m \neq a_{12}a_{13}a_{23}$, the shapes of the two dromions are changed after interaction. However, when parameter k_1 is selected as

$$k_1 = k_2\frac{a_{12} + a_{23} - a_{13} - a_{12}a_{13}a_{23}}{a_{12}a_{13}a_{23} + a_{23} - a_{12} - a_{13}} \tag{30}$$

the interacting constant m expressed by (10) equals $a_{12}a_{13}a_{23}$. In this case, the shapes of the two dromions do not change when they are interacting, but there is a phase shift determined by the interacting constants a_{ij}.

Figure 1 shows the interacting plots of two dromions, which are formed by three ghost line solitons about the physical field u, where the three solitons are determined by

$$\eta_1 = \tfrac{58}{23}x - \tfrac{195\,112}{12\,167}t \qquad \eta_2 = 2x - 8t \qquad \eta_3 = \tfrac{3}{2}y \tag{31}$$

and coupled coefficients a_{ij} are selected as

$$a_{12} = \tfrac{3}{2} \qquad a_{13} = \tfrac{1}{3} \qquad a_{23} = \tfrac{5}{2}. \tag{32}$$

In equation (31), k_1 comes from (30), according to equation (10), one may find $m = a_{12}a_{13}a_{23}$. Obviously, from figure 1, we can see that the shapes of the two dromions are unchanged after interaction.

Figure 2 also shows the interacting plots of two dromions for physical field u, where three solitons are formed by

$$\eta_1 = 2x - 8t \qquad \eta_2 = x - t \qquad \eta_3 = \tfrac{3}{2}y \tag{33}$$

and coupled coefficients a_{ij} are selected as

$$a_{12} = \tfrac{3}{2} \qquad a_{13} = \tfrac{4}{3} \qquad a_{23} = \tfrac{5}{2}. \tag{34}$$

According to equations (33), (34) and (10), we find $m = \frac{17}{9} \neq a_{12}a_{13}a_{23}$. From figure 2 we see clearly that the shapes of the two dromions are different before and after interaction.

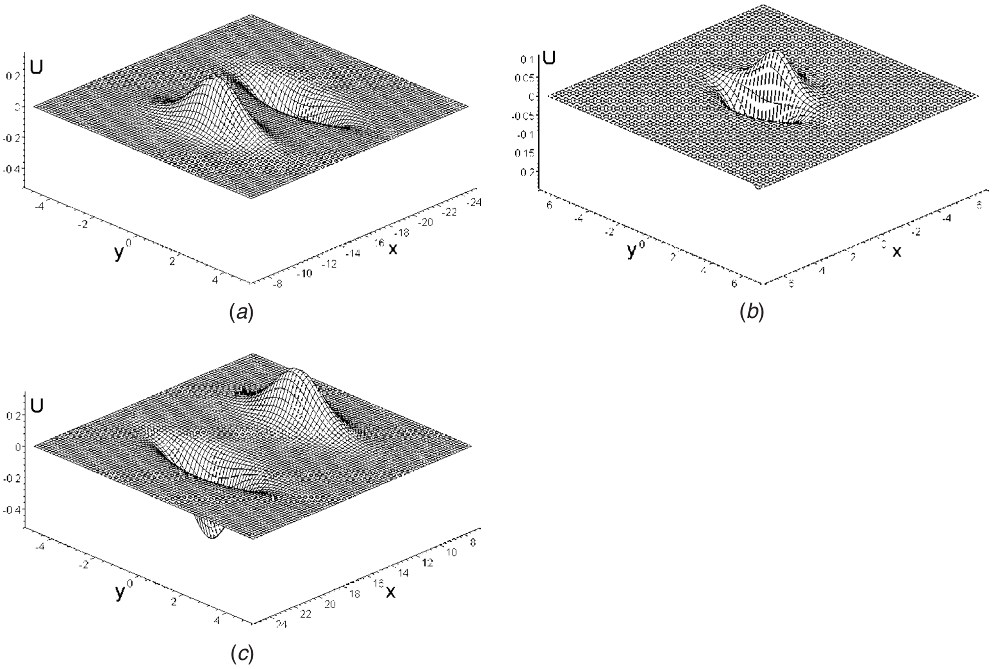

Figure 1. The interacting plots between two dromions about field u determined by equation (7) with (8)–(10) and (30) ($m = a_{12}a_{13}a_{23}$). The solitons are characterized by $\eta_1 = \frac{58}{23}x - \frac{195\,112}{12\,167}t$, $\eta_2 = 2x - 8t$, $\eta_3 = \frac{3}{2}y$. The time in the figures is: (*a*) $t = -3$, (*b*) $t = 0$, (*c*) $t = 3$.

3.2. Interaction between a line soliton and a y-periodic soliton

In this subsection, we will discuss the interaction between a line soliton and a y-periodic soliton about the field component v expressed by (13). When we assume that $k_1 > 0$, $k_3 > 0$ and $\omega_3/k_3 > \omega_1/k_1$, we obtain the expressions for a separated line soliton and a periodic soliton before and after interaction

$$f(\xi_1, \eta_1) = 1 + 2\exp(\xi_1)\cos(\eta_1) + a_{12}\exp(2\xi_1) \tag{35}$$

$$f(\xi_2, a_{13}) = a_{12}\exp(2\xi_1)\big[1 + a_{13}^2\exp(\xi_2)\big] \tag{36}$$

and

$$f(\xi_1, \eta_1, a_{13}) = \exp(\xi_2)\big[1 + a_{12}a_{13}^2\exp(2\xi_1) + 2a_{13}\exp(\xi_1)\cos(\eta_1)\big] \tag{37}$$

$$f(\xi_2) = 1 + \exp(\xi_2) \tag{38}$$

respectively, where subscripts 1 and 2 denote the coordinates of the y-periodic soliton and the line soliton, respectively. Based on the same reasoning as in section 3.1, we have only to consider the form of f. In a nutshell we have the following results,

$$[f_1(\xi_1, \eta_1), f_2(\xi_2 + 2\Gamma)] \longrightarrow \begin{cases} [f_1(\xi_1 + \Gamma, \eta_1), f_2(\xi_2)] & \text{for} \quad a_{13} > 0 \\ [f_1(\xi_1 + \Gamma, \eta_1 + \pi), f_2(\xi_2)] & \text{for} \quad a_{13} < 0 \end{cases} \tag{39}$$

where $\Gamma = \log|a_{13}|$. This expression shows that the phase shift due to the interaction is determined only by the coefficient a_{13}. The phase shift in the propagating direction is

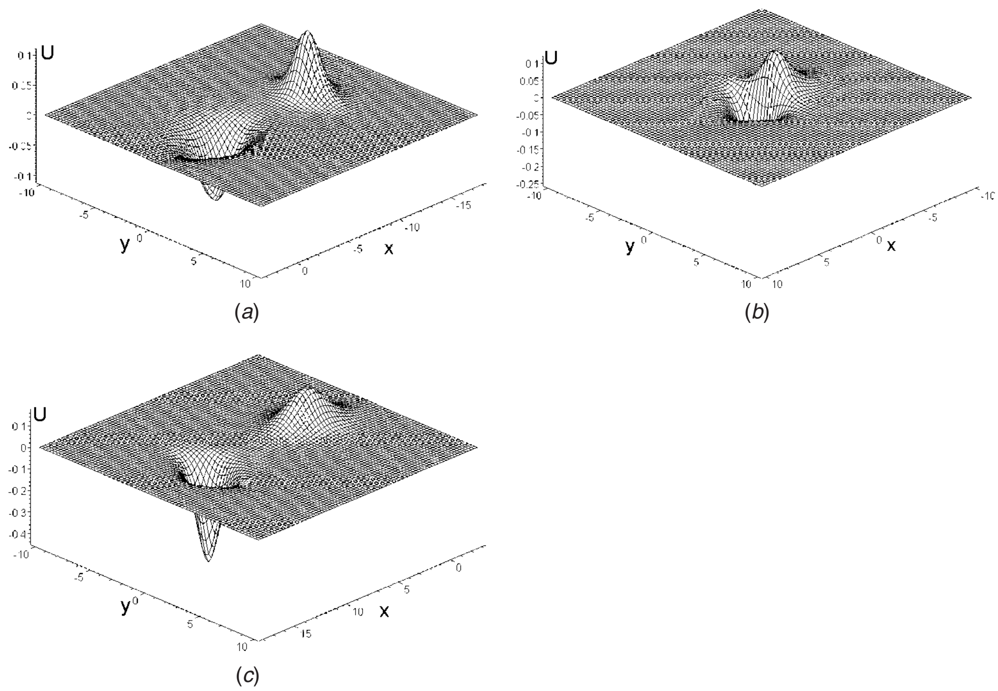

Figure 2. The interacting plots between two dromions about field u determined by equation (7) with (8)–(10) ($m \neq a_{12}a_{13}a_{23}$). The solitons are characterized by $\eta_1 = 2x - 8t$, $\eta_2 = x - t$, $\eta_3 = \frac{3}{2}y$. The time in the figures is: (a) $t = -3$, (b) $t = 0$, (c) $t = 3$.

determined by the magnitude of a_{13} while that in the transverse direction is determined by the sign of a_{13}. The sum of the phases in the propagating direction of the before-interaction solitons is twice that of the after-interaction solitons because the periodic soliton is madeup of two line solitons.

Figure 3 shows the interaction between a line soliton and a y-periodic soliton for $0 < a_{13} < 1$, where the function f is determined by (14), and

$$\xi_1 = \tfrac{3}{2}x - \tfrac{27}{8}t \qquad \xi_2 = 2x + \tfrac{1}{4}t \qquad \eta_1 = \tfrac{3}{2}y. \tag{40}$$

Because $0 < a_{13} \ll 1$, the interaction is the long-range repulsive interaction. As the line soliton is approaching the periodic soliton, the frontal part of the line soliton to the periodic soliton begins to grow, forming the hump on the peak, and the other part appears to be smaller. At the same time the trough of the periodic soliton increases, as a result, it looks fatter. While they keep a distance from each other, they seem to exchange energy and momentum through their tails in the propagating direction. Then they depart from each other recovering their original form.

3.3. Interaction between two y-periodic solitons

Now, we begin to discuss the interaction between two y-periodic solitons. If we assume that $k_1 > 0, k_2 > 0, \frac{\omega_{22}}{k_2} > \frac{\omega_{11}}{k_1}$ we obtain the expressions for two separated y-periodic solitons

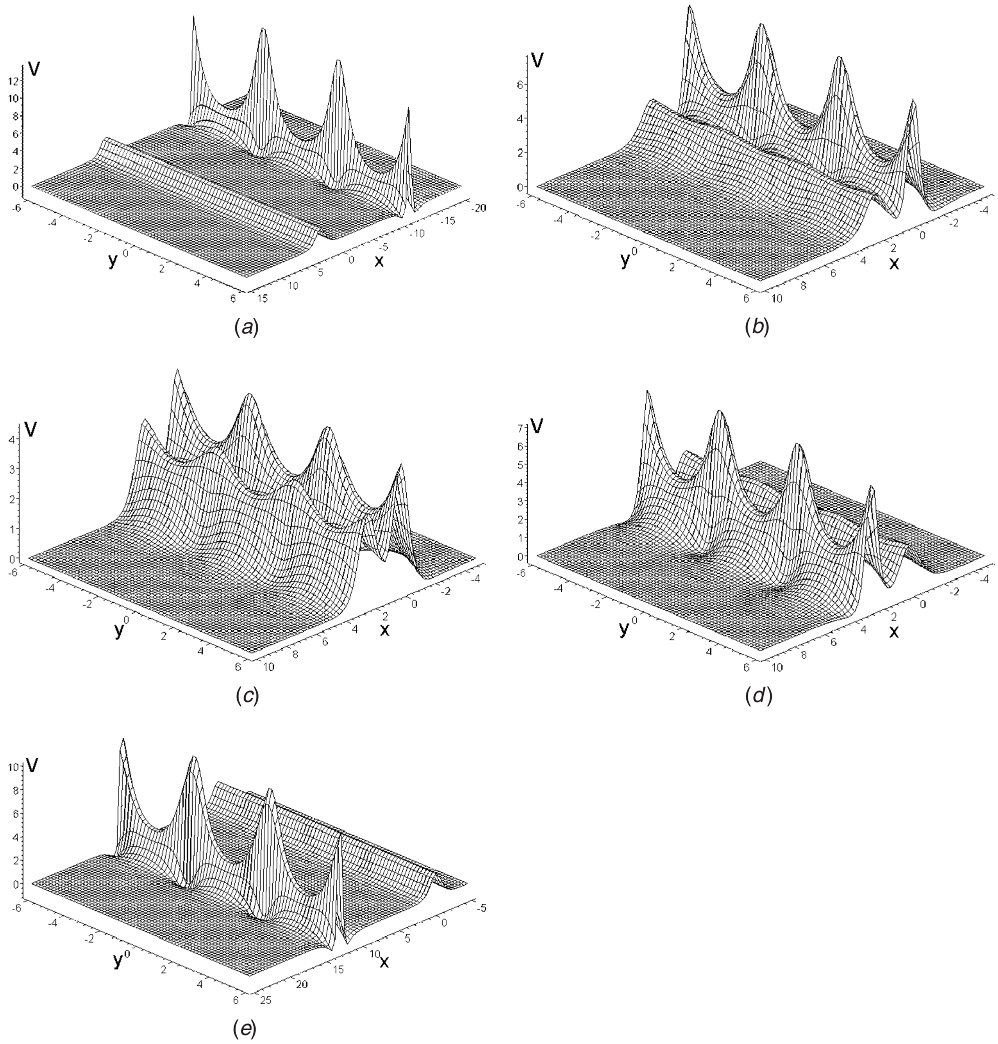

Figure 3. The interacting plots between a line soliton and a y-periodic soliton about field v determined by (13) with (14) and (15), where $(k_1, \delta_1, k_3) = (3/2, 3/2, 2)$; $a_{13} = 0.094\,34$ and $(\omega_3/k_3, \omega_1/k_1) = (1/8, -9/4)$; (*a*) $t = -5$, (*b*) $t = 0$, (*c*) $t = 0.2$, (*d*) $t = 0.5$, (*e*) $t = 5$.

before and after interaction

$$f(\xi_1, a_{13}a_{23}, \eta_1) = a_{34}\exp(2\xi_2)\big(1 + 2a_{13}a_{23}\exp(\xi_1)\cos(\eta_1) + a_{12}a_{13}^2a_{23}^2\exp(2\xi_1)\big) \qquad (41)$$

$$f(\xi_2, \eta_2) = 1 + 2\exp(\xi_2)\cos(\eta_2) + a_{34}\exp(2\xi_2) \qquad (42)$$

and

$$f(\xi_1, \eta_1) = 1 + 2\exp(\xi_1)\cos(\eta_1) + a_{12}\exp(2\xi_1) \qquad (43)$$

$$f(\xi_2, a_{13}a_{23}, \eta_2) = a_{12}\exp(2\xi_1)\big(1 + 2a_{13}a_{23}\exp(\xi_2)\cos(\eta_2) + a_{34}a_{13}^2a_{23}^2\exp(2\xi_1)\big). \qquad (44)$$

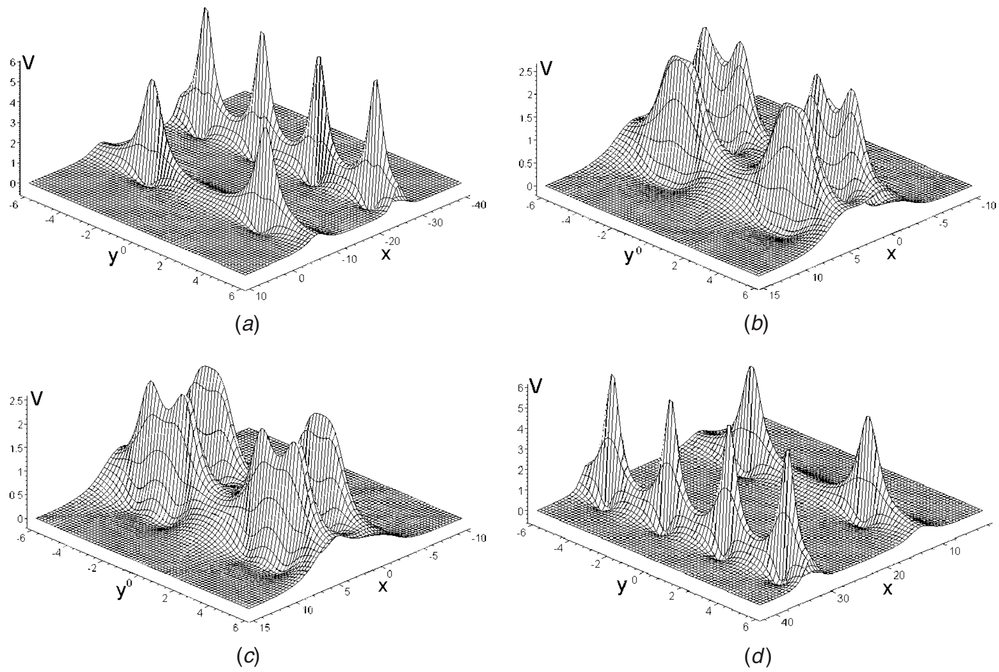

Figure 4. The interacting plots between two y-periodic solitons about field v determined by (19) with (20) and (21), where $(k_1, k_2, \delta_1, \delta_2) = (3/2, 1, 1, 2)$, $a_{13}a_{23} = 1/25$, $(\omega_{22}/k_2, \omega_{11}/k_1) = (1, 4/9)$; (*a*) $t = -25$, (*b*) $t = 1$, (*c*) $t = 2$, (*d*) $t = 30$.

For the same reason as in section 3.1 we have only to consider the form of f. We have the following results

$$[v_{p1}(\xi_1 + \Gamma, \eta_1), v_{p2}(\xi_2, \eta_2)] \longrightarrow \begin{cases} [v_{p1}(\xi_1, \eta_1), v_{p2}(\xi_2 + \Gamma, \eta_2)] & \text{for} \quad a_{13}a_{23} > 0 \\ [v_{p1}(\xi_1, \eta_1 + \pi), v_{p2}(\xi_2 + \Gamma, \eta_2 - \pi)] & \text{for} \quad a_{13}a_{23} < 0 \end{cases} \tag{45}$$

where

$$v_{pj} = 2(\log f_{pj})_{xx} \tag{46}$$

which is the y-periodic soliton solution and $\Gamma = \log |a_{13}a_{23}|$. This expression shows that the phase shift due to the interaction is determined only by the product $a_{13}a_{23}$. The phase shift in the propagating direction is determined by the magnitude of $a_{13}a_{23}$, while that in the transverse direction is determined by the sign of $a_{13}a_{23}$. The sum of the phases of the before-interaction soliton is equal to that of the after-interaction soliton as the usual soliton equations. It should be noted that the interaction term which is related to the phase shift is always positive in the KdV-type equation.

Figure 4 gives the interacting plots of two y-periodic solitons. In figure 4, $0 < a_{13}a_{23} < 1$, i.e. the case of repulsive interaction. The heights of the humps of both solitons are almost the same in this case. As two periodic solitons approach each other, two humps of the first soliton begin to move in the transverse direction so as to merge into one hump and every hump of the second soliton begins to separate in the transverse direction as shown in figure 4. While they keep a distance from each other, they seem to exchange energy and momentum through their tails in the propagating direction. Through this interaction the first soliton becomes the second soliton and the second soliton becomes the first soliton. Then they depart from each other recovering their original form. Note that they do not interpenetrate each other.

4. Summary and discussions

With the help of the bilinear method, we obtain some solutions of the ANNV equation which describes the interaction between two dromions for physical field u, the interaction between two y-periodic solitons and that between a line soliton and a y-periodic soliton for physical field v, respectively. We have discussed these interactions both analytically and graphically. For the first type of interaction, if the solution of the bilinear equation possesses the Hirota standard solution form including $m = a_{12}a_{13}a_{23}$ and $a_{ij} \neq 0$, field component $u = 2(\log f)_{xy}$ describes the elastic interaction, but when coupled coefficient $m \neq a_{12}a_{13}a_{23}$ $(a_{ij} \neq 0)$ u describes the inelastic interaction. It should be noted that coupled coefficients a_{ij} in the above discussion may be arbitrary constants which the equation allows. For the solution to describe the interaction between two y-periodic solitons is a new solution. It cannot be obtained from the standard Hirota bilinear method and the variable separation approach. However, it is the solution obtained by us using some special assumptions in the bilinear method. The interaction between two y-periodic solitons is related to the interaction coefficient product $a_{13}a_{23}$. The magnitude of $a_{13}a_{23}$ is related to the phase shift in the propagating direction and its sign is related to that in the transverse direction. So, in the case of $a_{13}a_{23} > 0$ for the ANNV equation, the phase shift in the transverse direction does not exist. We call the interaction attractive if $|a_{13}a_{23}| > 1$ is satisfied and repulsive if $|a_{13}a_{23}| < 1$. The long-range repulsive interaction is also discussed in this paper.

We know that the bilinear method and variable separation procedure are good approaches to find exact solutions of an integrable model. However, for some special types of equations such as the ANNV equation, NNV equation etc we cannot obtain special solutions and complex solutions from the standard Hirota bilinear method in which the coupled coefficients accord with the definition of Hirota and the variable separation procedure. Because there are fixed structures for some special soliton solutions some special assumptions can be used to obtain special solutions which are difficult to obtain using other methods. We hope these special procedures can be used to obtain more significant special solutions for more nonlinear equations. Whether the solution to describe the interaction between a y-periodic soliton and an algebraic soliton etc can be obtained via the procedure presented by us will be further studied.

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